

Structural Damage Detection Using Static Strain Data

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ABSTRACT: Structural health monitoring (SHM) is an effective way to maintain the civil infrastructure against unexpected collapse. In recent years, many researchers have developed numerous damage detection methods to identify possible damage locations. The damage locating vector (DLV) method proposed by Bernal has received significant attention because this method was verified by a series of proof-of-concept tests measuring accelerations. However, static strain measurements are more reliable and realistic than acceleration data in practice. In this paper, a strain DLV method, i.e. a method combining DLV and static strain measurements, has been developed. A series of numerical simulations and laboratory experiments have been conducted to verify the validity of the strain DLV method. From these results, the proposed strain DLV method is shown to determine damage locations successfully using a smaller number of strain sensors even when the strain at the damaged element is not available.

Keywords: Structural health monitoring, damage locating vector, static strain.

1 INTRODUCTION

Civil infrastructures like bridges are valuable assets of nation's economy and public safety. The construction of a long span bridge cost millions of dollars therefore maintenance of structural integrity is of utmost importance. More than 160,000 bridges are structurally deficient or functionally obsolete based on the 2004 report from the Federal Highway Administration. So far maintenance has been schedule-driven, or based on expensive and questionable visual inspection, sometimes resulting in false positive findings. A more affordable way to monitor the structural integrity of civil infrastructure is necessary for public safety and reliability.

One way is to employ a damage index which is a quantitative measure that gives a better view of the extent of the damage than response time histories. Many researchers have been employing the quantitative damage index methods to quantify the existence and severity of the damage. For example, Cawley and Adams [1] used the change of natural frequency, Wahab and Roeck [2] employed a mode curvature damage factor using modal updat-

ing, and Shi et al [3] introduced the ratio of change in model strain energy in each element as another damage indicator. In addition, Caicedo et al [4] proposed an index based on changes in the component transfer functions of the structure to quantify damage.

One of the most common types of damage is stiffness reduction due to steel corrosion or extensive concrete cracking. To find this type of damage, the most direct way is to employ the stiffness matrix or flexibility matrix. Pandey and Biswas [5] used changes in flexibility as a damage index, and later Bernal [6] proposed the damage locating vector (DLV) based on a dynamic flexibility matrix.

The DLV method is numerically and experimentally verified by Gao et al [7] using traditional accelerometers. Spencer and Nagayama [8] employed the DLV method for distributed computing strategies and verified this algorithm experimentally using wireless sensor network. Because this method is the only method verified experimentally with the state-of-the-art wireless sensor network,

this method is valuable to research for practical approach.

Practically, the strain measurement is one of the most reliable and available structural responses including acceleration and displacement. Displacement is usually hard to measure with appropriate accuracy without fixed reference points. To use acceleration data, the post-processing and system identification process is complicated and time consuming. Also, it is difficult to calculate mass normalization constants in the process of calculating the flexibility matrices. The expensive and complicated data acquisition systems and expensive sensors should be equipped. Static strain measurement can solve all these problems; the strain flexibility is the simplest measure once we have measured the strain – the minimal effort to calculate, no need to system matrices, conventional strain sensors and corresponding data acquisition systems are usually much more cheaper than those for acceleration measurements.

Two issues of the strain measurement should be solved; the number of necessary sensors for damage detection and the availability of the strain measurement at the damaged element. Installing strain gages in all elements is not efficient for two-fold; expensive cost and large amount of data. Next, the strain sensor at the damaged element is prone to be affected due to structural deficiency. In most cases, the strain at the damaged element contains direct information of local damage. Thus, the SHM algorithm with static strain employing a smaller number of sensors and having reliable performance without the information at the damaged element is necessary.

In this paper, a strain DLV method is proposed by combining DLV method with static strain measurement. The strain DLV approach determines the set of load vectors that span the null-space of the change in strain flexibility. These load vectors result in the same strain fields before and after damage. To obtain the same strain field even though the structure has damage, the strain DLVs do not induce strain at the damaged members. Using this concept, the elements, which have no internal force, when they are subjected to the strain DLV, are candidate damage elements. This method has been verified with a series of numerical simulations and laboratory experiments simulating moving load test measured with traditional strain gauges. From the results, the simplest post processing of the strain DLV method can make the process the least time-consuming. From the results, the performance of the strain DLV method has been verified to be efficient using a smaller num-

ber of sensors around the damage locations even when the strain sensor at the damaged element is not available. This method can be a reasonable quick-and-easy alternative method for the SHM tests for small and medium sizes of highway bridges for the field applications.

2 THEORETICAL BACKGROUND

2.1 Strain Damage Locating Vectors

The linear mapping matrix between load and strain is defined by the strain flexibility matrix such that:

$$\varepsilon = F^* \cdot L \quad (1)$$

The strain flexibility matrix is normalized strain with respect to a unit load. It is ideal to have a full strain flexibility matrix of a size equal to the number of DOFs in the structure. However, in practice a reduced strain flexibility matrix is constructed by measuring strains using a moving load.

Consider a system that can be treated as linear in the pre- and post-damage states as shown in Figure 1. The system has damaged and undamaged strain flexibility matrices at n sensor locations given by F_d^* and F_u^* respectively.

Assume that a number of load distributions exist to produce identical deformations when each load distribution is applied to the undamaged and damaged systems. If all distributions that satisfy this requirement are collected in the matrix L , one can write:

$$(F_d^* - F_u^*) \cdot L = \Delta F^* \cdot L = 0 \quad (2)$$

This relationship can be satisfied in two ways where either ΔF^* or L are not of full rank and L contains the vectors that span the null space. Since ΔF^* is nonzero when the system is damaged, the latter option is considered.

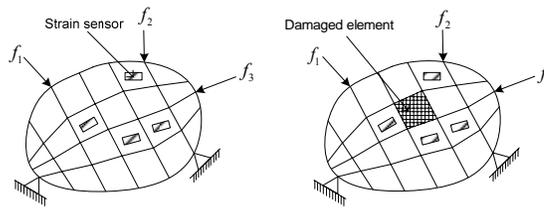


Figure 1. (a) undamaged system (b) damaged system

Performing singular value decomposition, the change of the strain flexibility matrix is expressed as:

$$\Delta F^* = [U] \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \approx 0 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}^T \quad (3)$$

where, U and L are unitary matrices, and S is a singular value matrix. s_2 consists of the singular values associated with the null space that will not be zero in actual application due to experimental noise and approximation. By post-multiplying the load vector matrix to the both sides of Equation(3):

$$\Delta F^* [L_1 \ L_2] = U \cdot \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \approx 0 \end{bmatrix} \cdot L^T \cdot L = [US_1 \ US_2] = [US_1 \ 0] \quad (4)$$

By direct comparison of Equation (4),

$$\Delta F^* L_2 = 0 \quad (5)$$

where, L_2 is the set of Strain DLVs that span to the null space of the change of the strain flexibility.

2.2 Implementation of Strain DLV method

Each of the strain DLVs is applied to the undamaged analytical model of the structure to calculate the deformation. The normalized accumulative stress of each member can be calculated as:

$$\bar{\sigma}_j = \frac{\sigma_j}{\max_k(\sigma_j)} \sigma_j = \sum_{i=1}^n \left(\frac{\sigma_j}{\max_k(\sigma_j)} \right) \quad (6)$$

Using the linear relationship between strain and stress, the corresponding strain is calculated. If an element has zero strain, then this element is a possible candidate of damage.

2.3 The proof of the Damage Locating Property

The proof of the Damage Locating Property of the DLVs is summarized in Bernal [6]. The main idea is that the strain DLVs are inducing zero stress in damaged elements so that the elements can be localized. For completeness of the research, the proof of this idea is summarized in this section.

Consider a linear structure discretized using n DOFs as shown in Figure 1. The m sensors are attached and k loads are applied in this structure. For convenience, the vector of DOFs is partitioned as $y = [y_a \ y_b]^T$ with $a = 1, \dots, k$ and $b = k+1, \dots, n$. The total potential energy for the system is

$$\Phi_{(y_a, y_b)} = U_{(y_a, y_b)} - W_{(y_a)} \quad (7)$$

where, U is the strain energy function and W is the potential energy of the loads. Assume that the displacements at the loaded points at equilibrium are known and constant. Careful inspection of Equation (7) shows that for this condition $W_{(y_a)}$ is a constant and Φ and U are reduced to functions of y_b . Applying the principle of stationary potential energy, one obtains:

$$\delta \Phi_{(y_b)} = \delta U_{(y_b)} = 0 \quad (8)$$

which shows that the strain energy is stationary at equilibrium. In addition, because the total potential energy is a minimum for equilibrium in Equation (7), $U_{(y_b)}$ is also a minimum. The theorem of minimum strain energy is then: *Of all the admissible strain distributions that yield the correct displacement at the loaded coordinates, the strain field that satisfies equilibrium minimizes the strain energy.*

Now consider the finite element model in Figure 1 before and after damage. According to the strain DLV procedure, the change of strain flexibility matrix has been calculated and the strain DLV matrix L has been identified. The domain of the structure can be subdivided into undamaged portions Ω_U and damaged portions Ω_D . The strain energy for the undamaged and the damaged states are:

$$U_U = \frac{1}{2} \left[\int_{\Omega_U} \varepsilon_u^T E_u \varepsilon_u dV + \int_{\Omega_D} \varepsilon_u^T E_u \varepsilon_u dV \right] = DLV_U^T \cdot y_z = q \quad (9)$$

$$U_D = \frac{1}{2} \left[\int_{\Omega_U} \varepsilon_d^T E_d \varepsilon_d dV + \int_{\Omega_D} \varepsilon_d^T E_d \varepsilon_d dV \right] = DLV_D^T \cdot y_a = q \quad (10)$$

where ε is the strain tensor, E is the strain to stress mapping matrix, and the subscripts u and d are the undamaged and damaged states, respectively.

The next step is to obtain an expression that is equal to or larger than U_D by invoking the minimum strain energy theorem. Next, ε_d in Equation (10) is replaced with any strain field that is geometrically admissible and leads to the correct deformation at the sensor locations. A member of the set of admissible functions is the undamaged strain field ε_u . Substituting this strain field into Equation (10) and expressing q by means of Equation (9):

Comment [S1]: Should you reference Theorem [7] here?

$$\int_{\Omega_U} \varepsilon_u^T E_u \varepsilon_u dV + \int_{\Omega_D} \varepsilon_u^T E_d \varepsilon_u dV \quad (11)$$

$$\geq \int_{\Omega_U} \varepsilon_u^T E_u \varepsilon_u dV + \int_{\Omega_D} \varepsilon_u^T E_u \varepsilon_u dV$$

or:

$$\int_{\Omega_D} \varepsilon_u^T E_d \varepsilon_u dV \geq \int_{\Omega_D} \varepsilon_u^T E_u \varepsilon_u dV \quad (12)$$

The stiffness over the damaged region can be expressed in terms of the undamaged stiffness as:

$$E_d = \alpha_{(x,y,z)} \cdot E_u \quad (13)$$

where α = a scalar, $0 \leq \alpha < 1$. Substituting Equation (13) into Equation (12) one gets:

$$\int_{\Omega_D} \alpha_{x,y,z} \varepsilon_u^T E_u \varepsilon_u dV \geq \int_{\Omega_D} \varepsilon_u^T E_u \varepsilon_u dV \quad (14)$$

which, given the fact that E_u is positive definite, can be satisfied only if the undamaged strain field is identically zero over the damaged region, thus completing the proof.

This proof is based on the assumption that the changes of the strain flexibility matrix from the undamaged and the damaged structures are all a result of stiffness reductions in the elements. By excluding the cases in which the damage causes an increase in the stiffness of the structures, this method can handle common damage types that result in stiffness reduction.

3 NUMERICAL SIMULATION

A 56-DOF statically indeterminate planar truss model shown in Figure 2 was examined for the numerical simulation. The Young's modulus and the cross sectional area of each element are 29,000 kips and 0.1359 in², respectively. The damage is assumed to be a 30% stiffness reduction to selected members. To obtain the 56 by 56 strain flexibility matrix, static strains from all elements according to all DOF's should be measured. However, measuring the strains of all the elements is impractical and applying forces to all elements is impossible in reality. Thus, the static strains from the 14 lower chords are measured according to a static input load from node 2 to node 14.

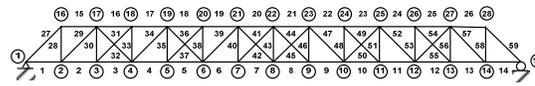


Figure 2. 2D planar truss model

After a series of DLV procedures, the normalized cumulative stresses when element 20 is damaged are calculated. To simulate the case when the strain at the damaged element is not performing properly, the strain measurement from the damage element is excluded deliberately. Zero stress at element 20 can show us that the element 20 can be a candidate damaged element.

This method is applicable also for the statically determinate structure; however, to localize the damage of the statically determinate structure, we definitely need the strain measurement at the damage element because no force redistribution takes place in this type of structure. Usually, most structures in reality are statically indeterminate to allow redundancy and a safety margin for the structure. Therefore, the Strain DLV method can be used for most field applications.

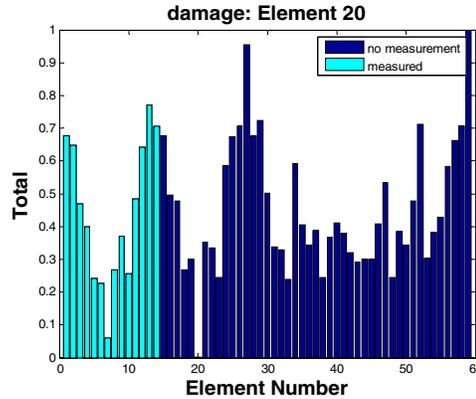


Figure 3 Normalized Cumulative Stress: Damage at Ele20

4 EXPERIMENTAL VERIFICATION

The **SDLV** method is experimentally verified by using a three dimensional truss structure at the Smart Structures Technology Laboratory of University of Illinois at Urbana-Champaign as shown in Figure 4. The details of this structure are summarized by Gao et al [7].

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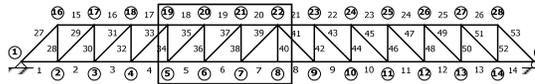


Figure 4 x-z plane view



(a) 3D Truss model (UIUC, SSTL)



(b) Supports (c) Moving weights

Figure 5 3D Truss model and Moving load system

To measure the strain flexibility effectively by simulating a moving truck load, a new loading system is designed and attached to the original structure. It consists of thirteen steel supports as shown in Figure 5 (b) with Thomson® 60 linear shafts and support rails. The loading system is connected with bolts to the original structure. The static weight is held by a Thomson® linear bearing in Figure 5 (c) so that the moving weights can move along the shaft from the far left node to the far right node smoothly.

Thirteen foil strain gages are installed on bars 6~8, 19~21, 36~42 in the box in Figure 4. Static strains are measured using National Instrument® Data Acquisition system with SCXI 1520 modules and SCXI 1314 boxes. The strain resolution is within 1 micro-strain for the NI-DAQ system. Then 80 lb of static load is applied to the joints of the lower chords using loading system in Figure 5.

The measured strain data from horizontal, diagonal, and vertical members are shown in Figure 6. The strain of the horizontal members is within 20 micro-strain, while those of the vertical and diagonal members are within 5 micro-strain. Here, the strain resolution of NI-DAQ may not be a significant source of error for horizontal members, but may be a significant source of error for the other members. The damage of the system is proposed to be a 40% stiffness reduction.

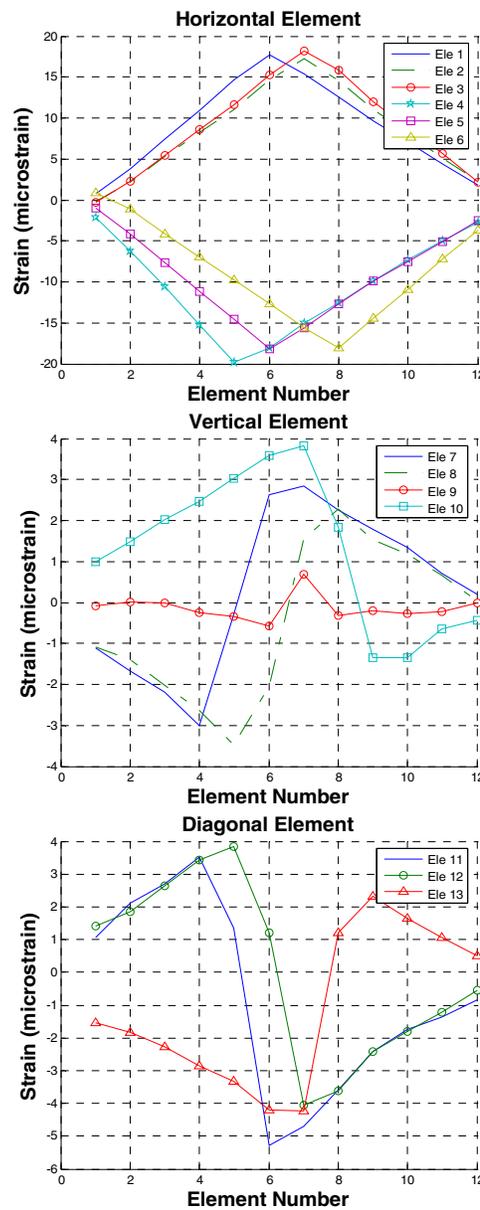


Figure 6 Measured Strain of Each Type of Members

CASE 1: DAMAGE TO ELEMENT 6

The experiment is conducted for undamaged and damaged system respectively. The first scenario is that element 6 is damaged. To simulate the situation when the strain measurement on element 6 is not available, data from element 6 is not used in the structural analysis process. In Figure 7, the strain at element 6 is low but not zero. Considering the noise level in the laboratory and errors, The threshold of strain can be set as 0.2 which means that the elements with stress levels less than 0.2 are regarded as candidate damaged elements.

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Then, element 6 and element 20 can be determined as possible candidates for damage. The low strain of element 20 is due to the truss geometry and force equilibrium.

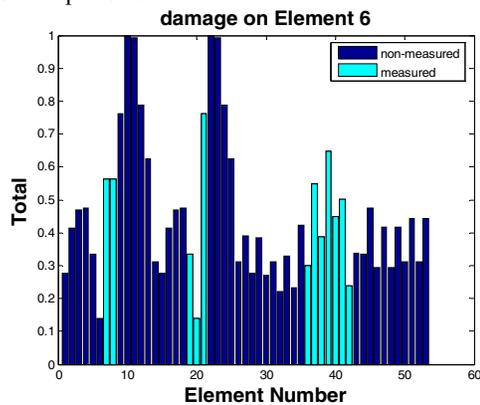


Figure 7 Normalized Accumulated Strain

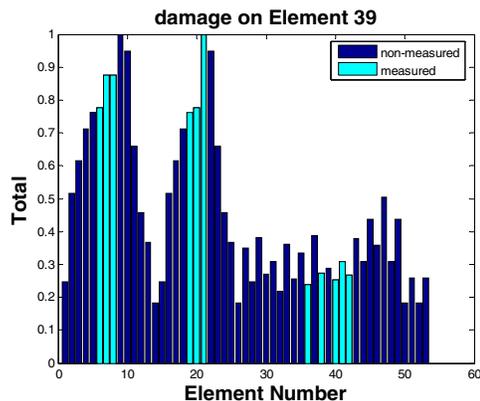
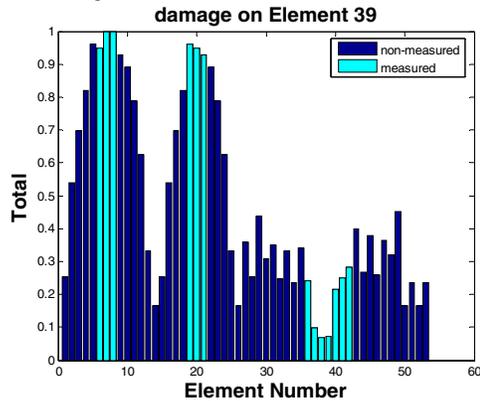


Figure 8 Normalized Accumulated Strain

CASE 2: DAMAGE TO ELEMENT 39

In this case, the diagonal element 39 is replaced by a member with a 40% stiffness reduction. The results are shown in Figure 8. For diagonal elements that are less stressed from the static loading, the change of strain flexibility is also very small so that the signal to noise ratio becomes smaller than

in case 1. Thus, without the measurement on the damaged element, damage location is uncertain. However, if we have measurement of element 39, the damaged element is found successfully.

One more noticeable item is strain resolution. In this experiment, the strain resolution is limited to 1 micro-strain because of data acquisition system. If the strain resolution can be improved significantly, for example, up to 0.001 micro-strain, the results from the strain DLV will be improved accordingly.

5 CONCLUSION

The strain DLV has been developed combining the DLV method and the static strain measurement. The performance of this method has been verified with numerical simulations and laboratory experiments using 2D and 3D planar truss models. From the results, the damaged elements are successfully localized using smaller number of strain sensors without measurements at the damaged elements. The experimental results show that the strain DLV should work better if we had better strain resolution from the appropriate data acquisition system.

Acknowledgement

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